Given that:

Assume that is a solution of , substituting into , we get:

Check for Wronskian determinant:

So, are linearly independence solutions of .

Thus, the general solution of the given differential equation is:

a) Given that:

Where:

Characteristic equation of the given ODE:

Since the right hand side of the given equation has three terms , and , therefore the particular solution also has three term: , respectively.

Solve fore from:

Since, is not a root of characteristic equation.

Hence, has the following form:

Solve fore from:

Since, is double root of characteristic equation.

Hence, has the following form:

Solve fore from:

Since, is not a root of characteristic equation.

Hence, has the following form:

So:

b) Given that:

Where:

Characteristic equation of the given ODE:

So, the complement solution is:

Since the right hand side of the given equation has two terms and , therefore the particular solution also has two term: , respectively.

Solve fore from:

Since, is a single root of characteristic equation.

So, has the following form:

Substituting into the equation we obtain:

Therefore:

Solve fore from:

Since, is not a root of characteristic equation.

So, has the following form:

Substituting into the equation we obtain:

Therefore:

So:

Thus, the general solution of the given differential equation is:

Differentiating both sides of , we get: .

Taking , we obtain:

Substituting into , it leads to:

Characteristic equation:

Therefore:

From :

Thus, the solution of the given system of differential equations is:

Given that:

Check for solution:

With , it holds that:. Substituting into , we get:

(valid)

With , it holds that:. Substituting into , we get:

(valid)

So, are solutions of

Check for linearity:

So, are linearly independence.

Thus, are linearly independence solutions of

Given that:

Assume that is a particular solution of , we have to find and .

Substituting into , we get:

Thus,

Solve for , we get the result:

From and initial condition :

i) So, the final result is:

ii) From :

Therefore:

The problem gives us , it leads to

So, the expression of is:

*(Too fucking long :) )*